Exercise 83

At what point on the curve $y = [\ln(x + 4)]^2$ is the tangent horizontal?

Solution

Take the derivative of the given function.

$$y' = \frac{d}{dx}[\ln(x + 4)]^2$$

$$= 2[\ln(x + 4)]^1 \cdot \frac{d}{dx}[\ln(x + 4)]$$

$$= 2\ln(x + 4) \cdot \left[\left(\frac{1}{x + 4}\right) \cdot \frac{d}{dx}(x + 4)\right]$$

$$= 2\ln(x + 4) \cdot \left[\left(\frac{1}{x + 4}\right) \cdot (1)\right]$$

$$= \frac{2\ln(x + 4)}{x + 4}$$

Since a horizontal tangent line has a slope of zero, set the derivative equal to zero and solve for x.

$$\frac{2\ln(x + 4)}{x + 4} = 0$$

$$2\ln(x + 4) = 0$$

$$\ln(x + 4) = 0$$

$$x + 4 = 1$$

$$x = -3$$

Find the y-coordinate corresponding to this value of x by plugging it in to the given equation.

$$y = [\ln(-3 + 4)]^2 = (\ln 1)^2 = 0$$

Therefore, the point on the curve $y = [\ln(x + 4)]^2$ that has a horizontal tangent is

$$(-3, 0).$$